Towards a Numerical Proof of Turbulence Closure

Subgrid Closure for Shell model of Turbulence using Machine Learning

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Giulio Ortali - g.ortali@tue.nl

Gianluigi Rozza - grozza@sissa.it

Federico Toschi - f.toschi@tue.nl

Alessandro Corbetta - a.corbetta@tue.nl

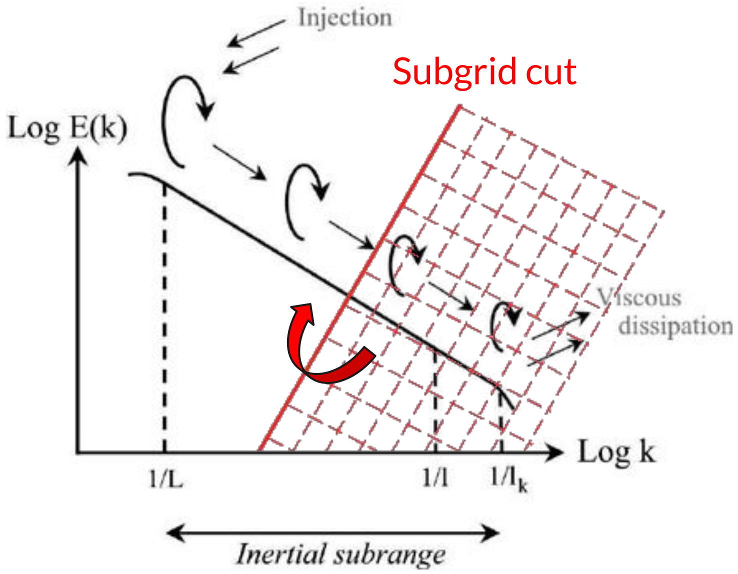
Outline

- ❖ Motivations & Problem Definition
- ❖ Shell Model Turbulence
- ❖ Methodology (LSTM-LES)
- ❖ Results
- ❖ Conclusion

Motivations: LES & Subgrid Closure

of DOFs of 3D turbulent flows grows as $\sim Re^{9/4}$

Idea: Use a **Subgrid Closure model** to decouple resolved/large dynamics from unresolved/small scale dynamics



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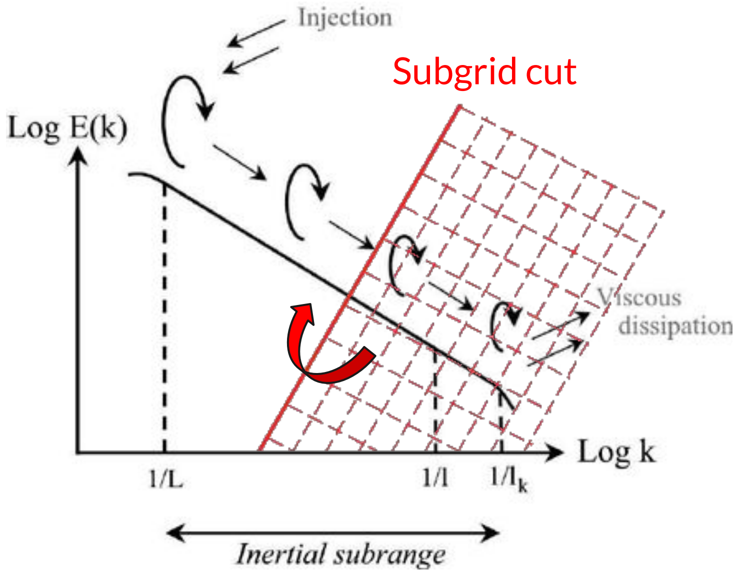
Many alternatives…

- ❖ Eddy viscosity
- ❖ Additional eqs. for the subgrid scale stress tensor
- ❖ …
- ❖ **Machine Learning based closures**



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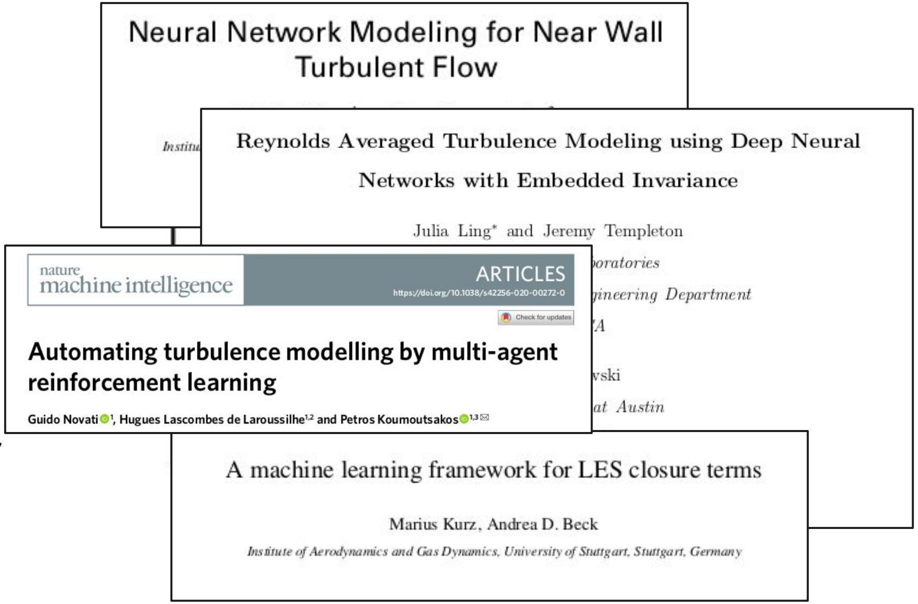

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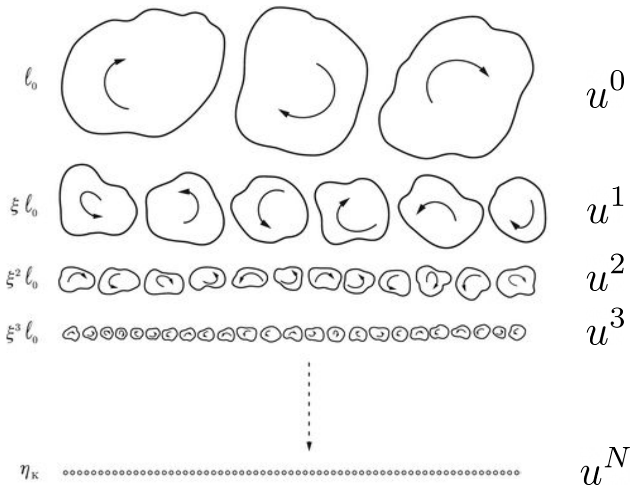
Reduced Setting: **Shell models of Turbulence**

Shell Models

System of N (~40) coupled ODEs representing **Homogeneous Isotropic Turbulence in Fourier Space**



total energy: $E = \sum_n |u_n|^2$

u_n: energy of fluctuations present at logarithmically equispaced scales $k_n = k_0 \lambda^n \quad (k_0 = 1, \lambda = 2)$

$$\frac{du_n}{dt} = F(u_{n-2}, u_{n-1}, u_n, u_{n+1}, u_{n+2}) - \nu k_n^2 u_n + f_n$$

- $F(u_{n-2}, u_{n-1}, u_n, u_{n+1}, u_{n+2})$: Convective term, coupling the shells. **nonlinear** and **local** (in fourier space)
- $\nu k_n^2 u_n$: Dissipation, only present at the smallest scales
- f_n: Forcing, active only at large scales $f_n = f_0 \delta_{n,0}$

Shell Models: Subgrid Closure

1. Split in "**large scales**" $u^<$ and "**small scales**" : $u^>$

$$u = \{u_n\}_{n=0}^{N} = \begin{bmatrix} u^< \\ u^> \end{bmatrix} = \begin{bmatrix} \{u_n\}_{n=0}^{N_{cut}} \\ \{u_n\}_{n=N_{cut}}^{N} \end{bmatrix}$$

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This problem is **not trivial** even in the simplified setting of the Shell Models, and has been recently tackled in [4]

Re	10^{12}
N_{cut}	15
N_η	30
Δt_{DNS}	10^{-8}
Δt_{LES}	10^{-5}

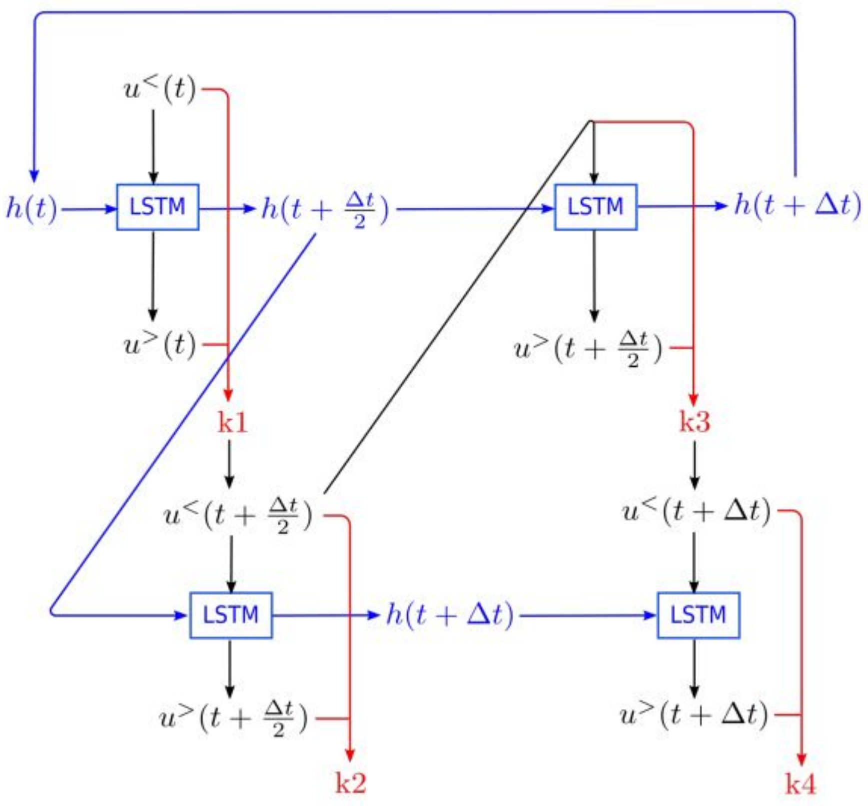
[4] L. Biferale, A. A. Mailybaev, and G. Parisi, **Optimal subgrid scheme for shell models of turbulence**, Phys. Rev. E 95 (2017).

Methodology: LSTM-LES

- Modification of classical **Runge-Kutta** multistep integrator:

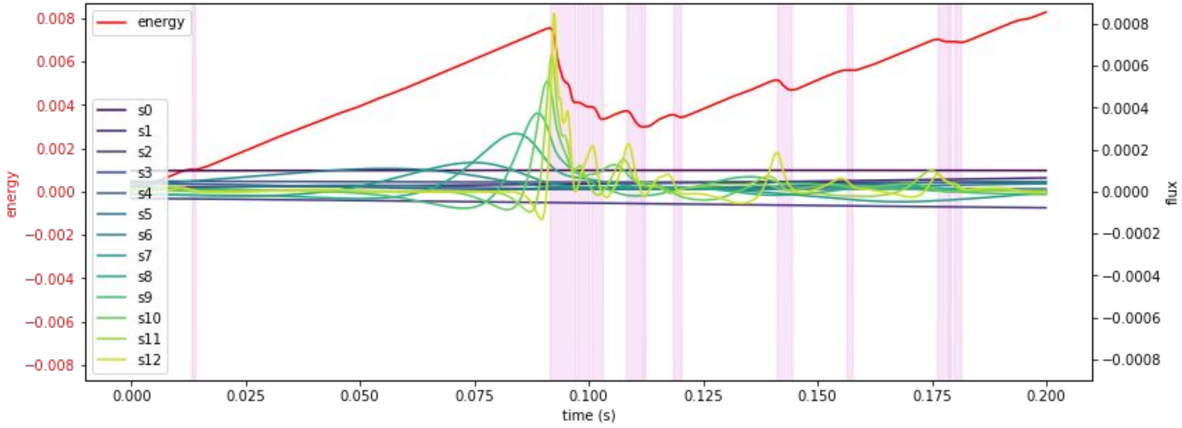
$$u^{<}(t+\Delta t) = u^{<}(t) + \frac{\Delta t}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4).$$

- at each sub-step, we augment with information regarding the **subgrid fluxes**
- The subgrid fluxes are computed via Recurrent **Long-Short Term Memory** Neural Networks



Results

❖ Energy balance for the Fully Integrated shell model



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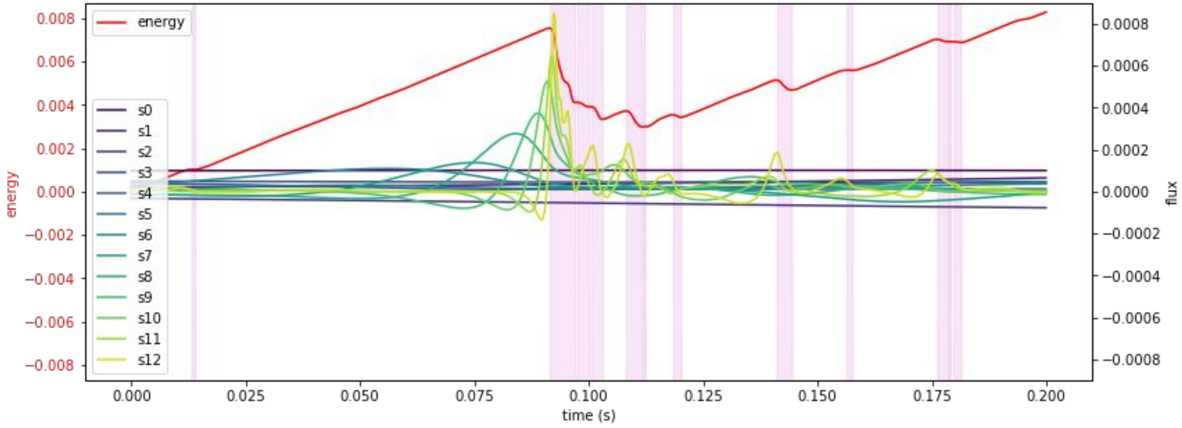
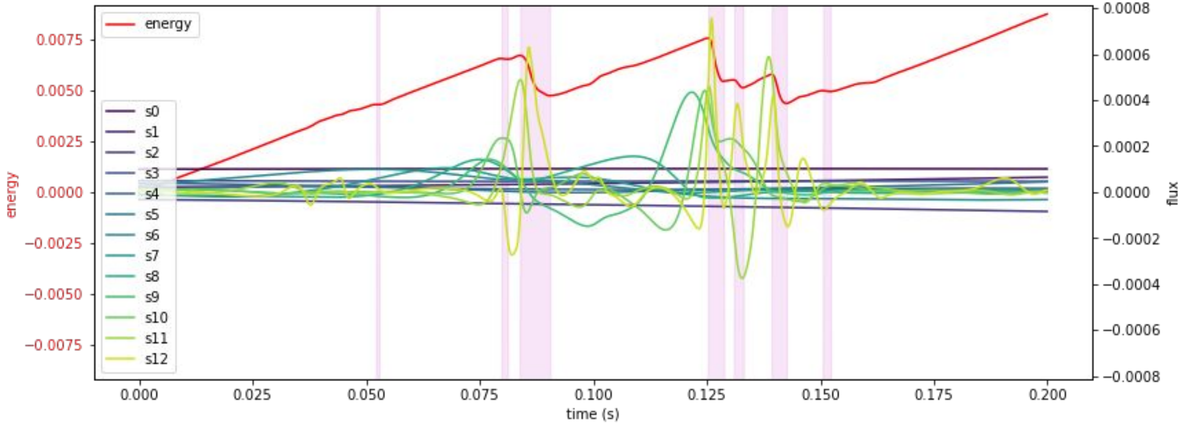


❖ Energy balance for our LSTM-LES

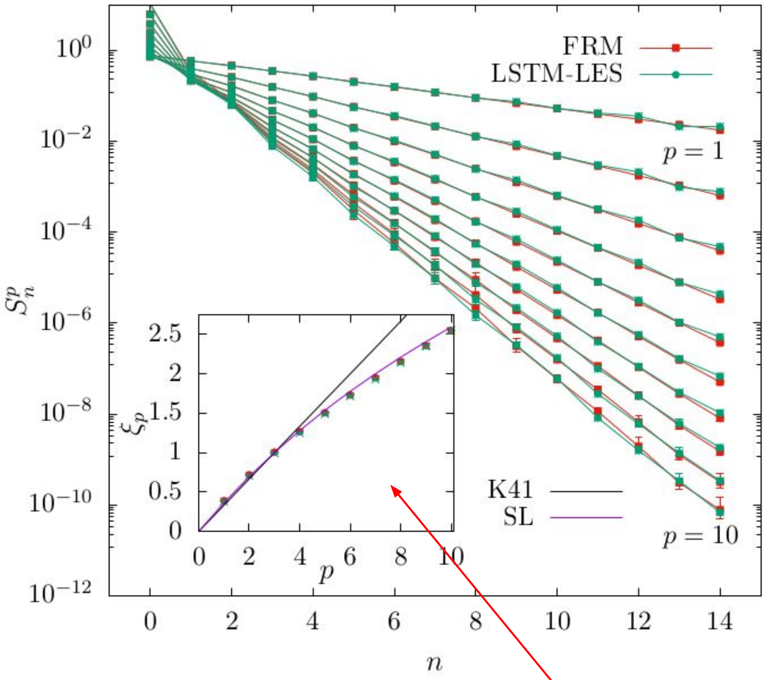


Results

Eulerian Structure Functions, red for the fully resolve, green for our LSTM-LES

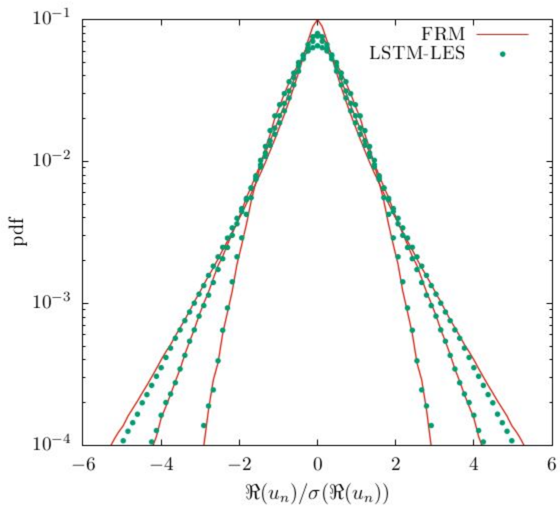
- shell index in the x-axis
- subgrid cutoff at n=15
- orders p=1..10

On the inset, **Anomalous exponents**



$$S_n^p = < |u_n|^p > \sim k_n^{-\xi_p}$$

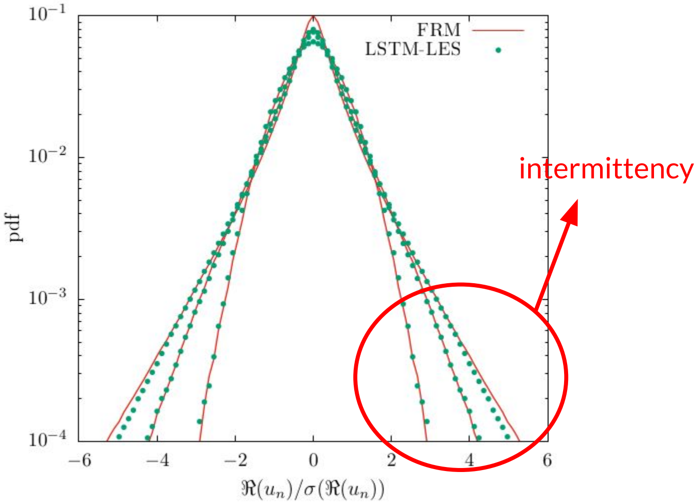
Average over time and samples



(Normalized) pdf of the (real part) of the shells, red for the fully resolved, green for our LSTM-LES

$$\Re(u_n)/\sigma(\Re(u_n))$$

- shown shells n=4, 9 and 14 (cutoff is at 15)
- Normalized log scale
  (Gaussian is parabola)



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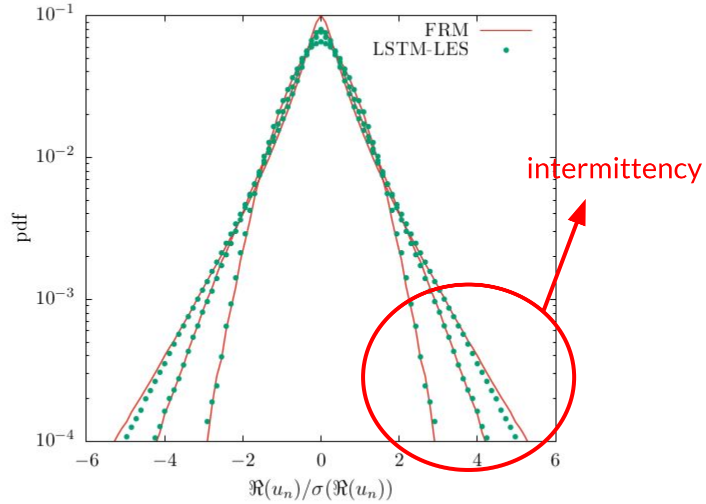

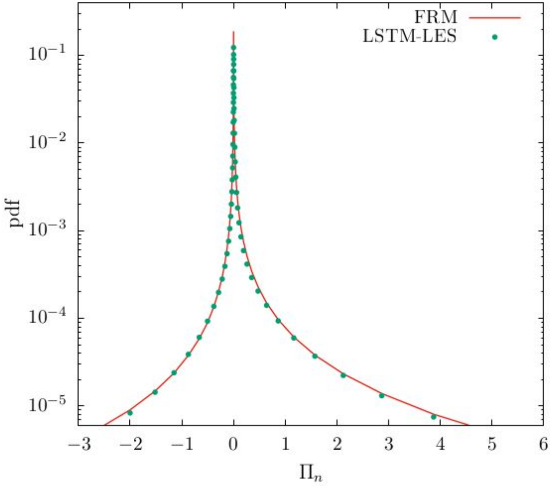

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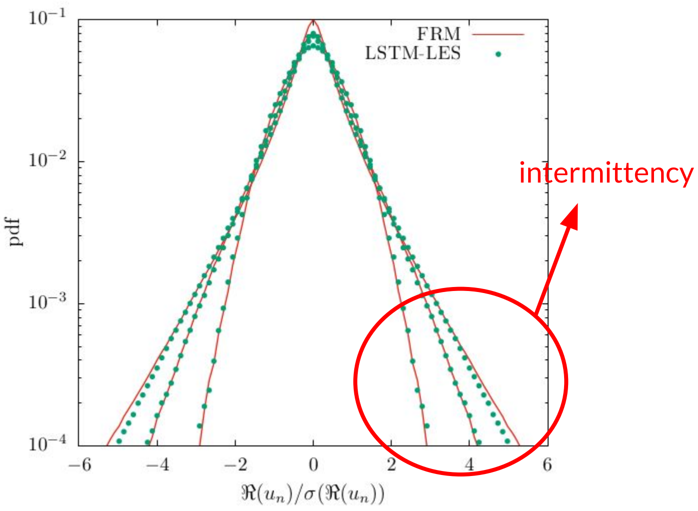
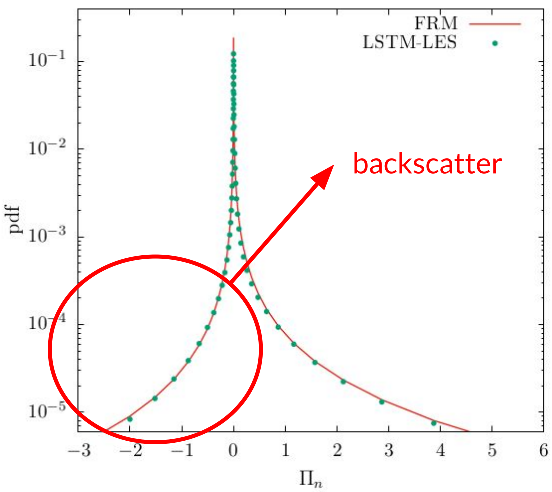
Convective Fluxes, **red** for the fully resolved, **green** for our LSTM-LES

$$\Pi_n = \frac{d}{dt}E_n = \frac{d}{dt}\sum_{m=0}^{n}|u_m|^2$$

- shown shell n=12
- log scale





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Conclusions & Perspectives

- Our approach reproduces the statistics of the filtered fully resolved model up to very **high order**
- Physical features such as **backscatter** and **intermittency** are correctly reproduced
- Extension of the proposed model to **real 3D Homogeneous Isotropic Turbulence** is not trivial...

$$\frac{du_n}{dt} = F(u_{n-2}, u_{n-1}, u_n, u_{n+1}, u_{n+2}) - \nu k_n^2 u_n + f_n$$

We lose the hypothesis of locality of the convective term

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Thank you for the attention

References

- [1] G. Ortali, A. Corbetta, G. Rozza, and F. Toschi, **Towards a Numerical Proof of Turbulence Closure,** arxiv (2022)
- [2] U. Frisch, **Turbulence: The Legacy of A. N. Kolmogorov** (Cambridge University Press, 1995).
- [3] L. Biferale, **Shell Models of Energy Cascade in Turbulence**, Annu. Rev. Fluid Mech. 35, 441 (2003).
- [4] L. Biferale, A. A. Mailybaev, and G. Parisi, **Optimal subgrid scheme for shell models of turbulence,** Phys. Rev. E 95 (2017).

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