Structure-Preserving Neural Networks for Hamiltonian Systems

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Hamiltonian Neural Networks

HNNs can be used to predict trajectories of Hamiltonian systems. They learn a separable Hamiltonian and embed a symplectic numerical integrator inside their topology.

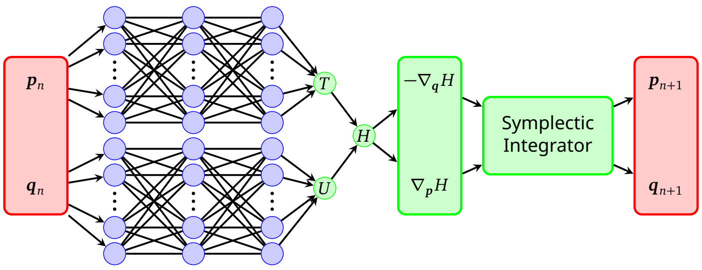

Figure 1: Hamiltonian neural network by Chen *et al.* [1].

SympNets

Alternatively, SympNets directly focus on learning a symplectic map (like the flow map of a Hamiltonian system), without the need to learn a separable Hamiltonian.

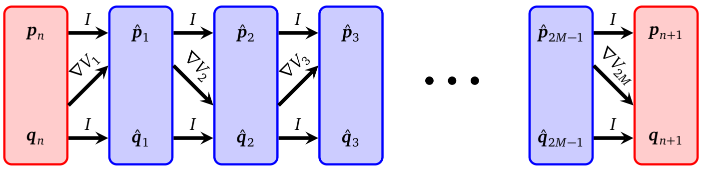

Figure 2: A standard SympNet by Jin *et al.* [2].

However, every two layers learn a hidden Hamiltonian. Hence, a SympNet is equivalent to multiple HNNs in series. This yields a new visualization displayed in Figure 3.

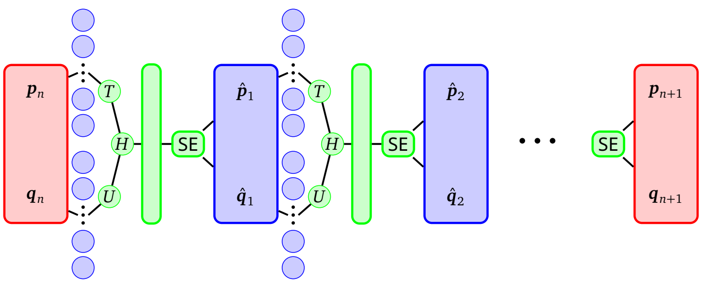

Figure 3: New point of view on the topology of SympNets.

Mathematics and Computer Science

New symplectic NNs

The new point of view on SympNets motivates a general framework of NNs for Hamiltonian systems. This framework includes HNNs and SympNets as two edge cases. The more flexible framework allows the creation of NNs that are superior to HNNs and SympNets.

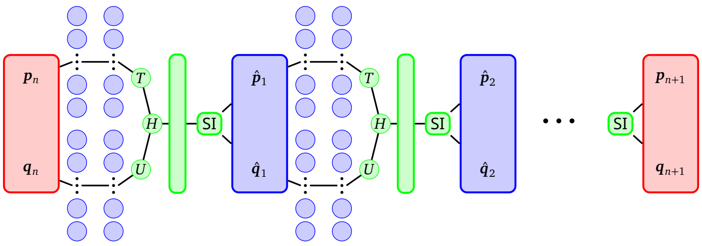

Figure 4: An improved symplectic NN.

Results

The performance of these NNs is analyzed on data of the 3-body problem. While the NNs are only trained on data until $t_{\text{max}} = 4.5$, they are used to predict until $t_{\text{max}} = 8$.

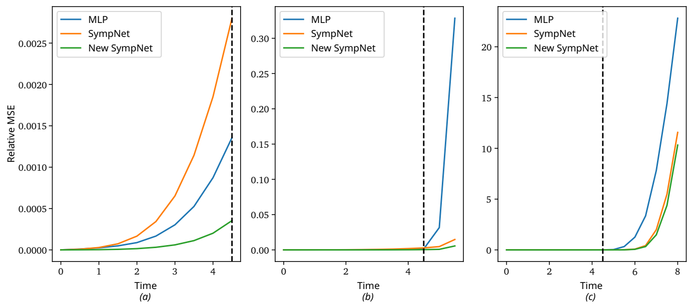

Figure 5: Comparison of different NN topologies.

The SympNet is merely more robust than the physics unaware MLP. However, the new symplectic NNs are more accurate inside the training data and outside.

References

[1] Z. Chen *et al.*, Symplectic recurrent neural networks, *8th International Conference on Learning Representations*, ICLR, 2020.

[2] P. Jin *et al.*, SympNets: Intrinsic structure-preserving symplectic networks for identifying Hamiltonian systems, *Neural Networks*, Vol. **132**, 2020.